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Determining Gravitational Acceleration with the Mathematical Pendulum Swing Method

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ABSTRACT

Pendulum-derived gravitational acceleration values vary unpredictably due to unquantified string length and initial angle dependencies. This study aims to assess the feasibility and precision of the mathematical pendulum swing method in measuring gravitational acceleration. The research investigates the relationship between the pendulum's string length, oscillation period, and initial deviation angle. It is hypothesized that longer strings will result in slower oscillations while shorter strings produce faster motions. Additionally, the study examines how these factors collectively impact gravitational acceleration measurement. Through controlled experiments, the study's goal is to determine the precision of the pendulum swing method in determining gravitational acceleration and the factors influencing its reliability. Results demonstrate that a simple mathematical pendulum accurately determines gravitational acceleration with a calculated value of approximately 9.95974m/s². The findings confirm that the square of the oscillation period is directly proportional to the string length ($T^2 \propto l$) and emphasize the importance of maintaining the initial deviation angle under 10 degrees to ensure measurement accuracy. These findings reinforce the pendulum's utility in physics education and affordable gravitational field assessment, particularly in resource-limited settings. The study provides a foundational framework for improving experimental accuracy in classical mechanics and highlights the method's enduring relevance in scientific pedagogy and fundamental research.

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INTRODUCTION

The pendulum motion is classified as a simple harmonic rotational motion with a small angle of deviation, so it can be implied that $\sin\theta \approx \tan\theta \approx \theta$. This motion is caused by the torque generated by the gravitational force of the object. Simple harmonic motion itself is a back and forth motion through an equilibrium point with a fixed period. Meanwhile, rotational motion is the movement of an object that rotates on its axis. Simple harmonic rotational motion is characterized by a back and forth motion that follows a path in the form of part of a rotation, with a fixed period (Yuningsih et al., 2020).

Gravitational acceleration refers to the acceleration an object experiences due to Earth's gravitational force (Neslušan, 2023). The standard gravitational acceleration on Earth's surface is approximately 9.81 m/s². However, this value can vary slightly depending on geographic location and altitude, influenced by differences in Earth's mass distribution and the effects of its rotation. In physics, gravitational acceleration is commonly used to analyze the motion of freely falling objects (Silvi Mutia et al., 2024).

In the study of dynamical systems, the mathematical pendulum is a classic model that is essential for understanding harmonic behavior and oscillatory motion (Rini & Saefan, 2023). It consists of a mass attached to a string, allowing it to swing freely under the influence of gravity. The motion of a mathematical pendulum can be described by simple harmonic equations when the restoring force is proportional to the displacement from the equilibrium position (Sudarmanto et al., 2023). After being shifted sideways from the equilibrium (center) point by the force of gravity, the pendulum will swing in the vertical plane (Ismail, 2020). The swing will be able to determine the amount of time (period) required by the weight to cause a vibration (Fauzi et al., 2024).

Previous research on pendulum systems has extensively explored the effects of the initial deviation angle (θ_0) and rope mass on pendulum dynamics. Studies such as those by Kidd & Fogg (2002) and Nelson & Olsson (1986) have shown that large initial angles introduce nonlinearities, increasing the period beyond the small-angle approximation ($T \approx 2\pi\sqrt{(L/g)}$), while research by Lekner (2004) and Pujol (2010) demonstrated that a massive rope alters the effective length and period due to distributed mass. However, most studies have examined these factors in isolation, leaving a research gap in understanding their combined effects, which is crucial for real-world applications like crane cables and swinging bridges. This study aims to bridge that gap by investigating the interactive influence of θ_0 and rope mass on pendulum motion, developing a corrected mathematical model, and validating findings through experiments and simulations (Kidd & Fogg, 2002; Lekner, 2004; Nelson & Olsson, 1986; Pujol, 2010).

Amedeker MK carried out a mathematical pendulum experiment and the result was that the gravitational acceleration in the experiment was g = 9.93m/s², which was 1.2% greater than the universal value (g = 9,81m/s²) (Amedeker, 2022). Moreover, a similar study was conducted by Oliveira V, yielding an accurate gravitational acceleration value of g=9.81 m/s² after accounting for the mass of the string. Therefore, this study aims to examine the accuracy and feasibility of a mathematical pendulum in measuring gravitational acceleration (Oliveira, 2016). The study focuses on experimenting the relationship between pendulum length and oscillation period. The effect of the initial deviation angle is also studied. The research will be able to evaluate how gravitational acceleration is influenced by these factors. The researchers hope to provide theoretical and practical insights in the field of physics regarding pendulum dynamics.

This study aims to evaluate the accuracy and feasibility of using the mathematical pendulum swing method to measure gravitational acceleration, while explicitly assessing how pendulum length, initial angular deviation, and neglected string mass influence the outcomes. By addressing these underexplored variables, the study seeks to contribute both theoretical

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understanding and practical refinement of gravitational measurement methods in physics education and experimentation.

The hypothesis of this study states that the pendulum string's length significantly affects both the measurement of gravitational acceleration and its motion. Particularly, it is suggested that a shorter string will result in a faster pendulum motion, whereas a longer string will result in a slower pendulum motion. Furthermore, it is believed that the acceleration caused by gravity is influenced by the pendulum's swing period and string length. Additionally, it is expected that the length of the string would affect the distance of the pendulum's track. This also will affect the dynamics of the system as a whole. All of these theories aim to better understand the factors affecting pendulum behavior in experimental settings.

Gravitational acceleration plays a critical role not only in theoretical physics but also in various practical and interdisciplinary applications, such as satellite navigation systems, seismic monitoring, and structural engineering. Accurate measurement of this fundamental constant is essential for both scientific understanding and real-world implementations. Thus, simple yet reliable experimental approaches like the mathematical pendulum become valuable tools not only in educational settings but also in broader geophysical and engineering contexts.

METHODS

Materials and Equipment

The materials used in this study included a stopwatch to measure time intervals accurately, two rulers for length and angle measurements, a plasticine mass to serve as the pendulum bob, and a string approximately 40 cm long. A flat elevated surface, such as a table, was used to support the setup, and a heavy object like a book was employed to stabilize the apparatus during the experiment.

Experimental Setup

The experimental setup began by securing a ruler to the edge of a table with a heavy object, which acted as the fixed pivot point. One end of the string was tied to the ruler, and the other to the plasticine mass to create the pendulum. The length of the pendulum was defined as the distance from the pivot to the center of the pendulum bob. The second ruler was positioned vertically to help control and maintain the initial angular displacement, ensuring uniform conditions across trials.

The shape of a simple mathematical pendulum circuit is shown in Figure 1:

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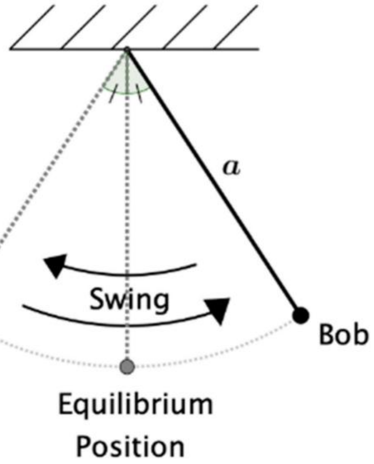

Figure 1. The Shape of a Basic Mathematical Pendulum Circuit (Rivera-Figueroa & Lima-Zempoalteca, 2021)

Procedure

Prior to each trial, the height of the table and the distance from the bob to the floor were measured for consistency. The pendulum was displaced at a small angle—less than 10 degrees—to satisfy the small-angle approximation and then released to oscillate freely. Using a stopwatch, the time required to complete ten full oscillations was recorded. This measurement was repeated for various string lengths while keeping the bob mass and deviation angle constant. By systematically changing the string length and recording the corresponding oscillation periods, the relationship between pendulum length and oscillatory motion was evaluated, and the gravitational acceleration was subsequently calculated.

Variable Control

In this study, the string length served as the independent variable and was systematically adjusted throughout the experiment. The oscillation period was the dependent variable, measured in each trial. The mass of the pendulum bob and the initial deviation angle were controlled to ensure experimental consistency. Measuring the time for ten complete oscillations helped reduce timing errors and provided more accurate data for analysis.

RESULTS

Table 1. Observation Results Table

No	Load distance from the floor	String length	Period (T)	Quadratic period (T^2)
1	0.04 m	0.36 m	1.217 s	1.48109 s^2
2	0.08 m	0.32 m	1.146 s	1.31332 s^2
3	0.13 m	0.27 m	1.054 s	1.11092 s^2
4	0.16 m	0.24 m	0.989 s	0.97812 s^2
5	0.20 m	0.20 m	0.911 s	0.82992 s^2
6	0.25 m	0.15 m	0.803 s	0.64481 s^2
7	0.28 m	0.12 m	0.726 s	0.52708 s^2

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The experimental results, summarized in Table 1, demonstrate the relationship between string length and oscillation period for a simple pendulum. As the string length decreased from 0.36 m to 0.12 m, the measured period (T) systematically declined from 1.217 s to 0.726 s, while the corresponding quadratic period (T^2) decreased from 1.48109 s^2 to 0.52708 s^2.

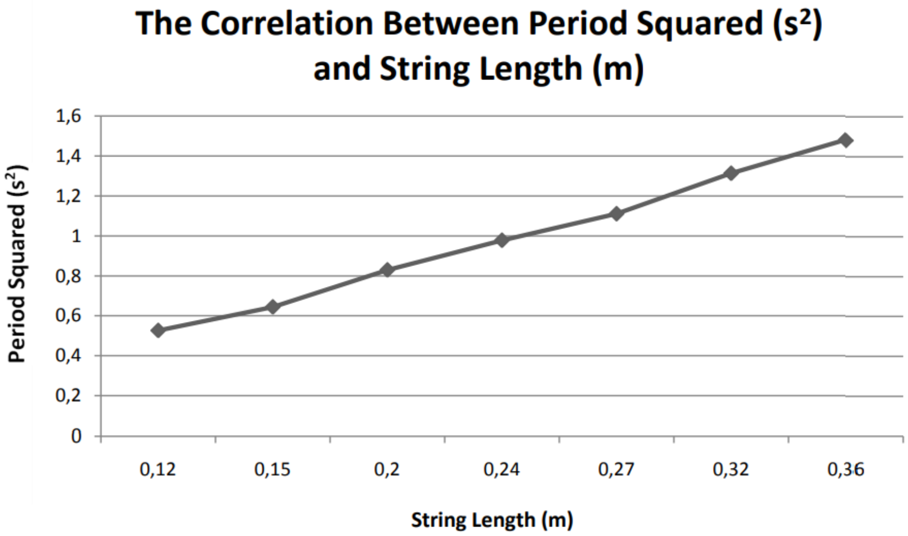

Figure 2. Graph of Observation Results

The relationship between the period squared (T^2) and string length (L) of the pendulum is illustrated in Figure 2. The graph shows a clear linear trend, where T^2 increases proportionally with L, consistent with the theoretical prediction for a simple pendulum governed by the equation $T^2 = (4\pi^2/g)L$.

Discussion

Angular Simple Harmonic Motion refers to oscillatory motion along a semicircular path, exemplified by the behavior of a mathematical pendulum (Pratidhina et al., 2020). However, true simple harmonic motion is rarely observed in real-world scenarios due to the presence of external forces, such as air resistance or friction, which alter the system's equation of motion and cause changes in amplitude over time (Wardani, 2020). Despite these practical limitations, the motion of a pendulum can be approximated as simple harmonic motion, particularly for small angular displacements (typically $\leq 15°$), where it swings periodically with a consistent time interval. In this context, the restoring force responsible for the pendulum's motion is the component of the gravitational force acting tangentially along the arc of its trajectory, as described by the relevant physical equation (Sardjito & Yuningsih, 2020):

$$F = mg \sin \theta$$

F is the restoring force that keeps the pendulum swinging continuously, and the negative symbol is present in simple harmonic motion because the acceleration vector's orientation is opposed to the displacement vector (Gea et al., 2022). These are the force elements influencing the system of pendulums.

The restoring force is a force that is always in the opposite direction of the deviation (position) and whose magnitude is proportionate to the deviation. The pendulum's period value can be expressed using the following equation if the angle θ is sufficiently small (θ ≤ 10˚) (Mohammad Bakhit Mufti et al., 2022).

$$T=2\pi\ ((L/g)^2)$$

T = Period of simple harmonic motion of a mathematical pendulum

g = Acceleration due to gravity on the Earth's surface

l = Length of the mathematical pendulum's string

This equation can be used to determine the relationship between the length of the string used, the acceleration of gravity on the surface of the earth, and the period of the mathematical pendulum. To make the computation easier, we suppose that the object utilized in this research is a mathematical pendulum, causing simple harmonic motion to occur.

From Table 1, it can be observed that the results align with the theoretical expectation that the longer the string, the greater the resulting period, and conversely, the shorter the string, the smaller the period. The consistent reduction in both T and T^2 across all trials suggests a predictable dependence of the period on string length, supporting the classical pendulum model. These findings provide an empirical basis for further analysis, such as deriving gravitational acceleration or examining the validity of the small-angle approximation in the experiment. The squared values of the period and the string length obtained from each observation are subsequently analyzed through a graphical representation of the correlation between the period squared and string length.

The graph of T^2 (period squared) against the length of the string represents a linear function with a constant gradient shown in Figure 2, which corresponds to the acceleration due to gravity at the Earth's surface. From the experimental data, the gradient of the graph yielded a value of approximately 9.95974 m/s² for gravitational acceleration. However, this result deviates from the accepted value of 9.807 m/s², and several factors contribute to this discrepancy. First, the presence of air resistance (an external force) causes the simple harmonic motion to transition into damped oscillatory motion, altering the pendulum's equation of motion. Second, in real-world conditions, the mass of the string cannot be neglected, as it plays a significant role in determining the pendulum's dynamics. The assumption of a massless string introduces inaccuracies in the calculation of gravitational acceleration. Additionally, human errors, such as inaccuracies in measuring the string's length, the load's distance, and the oscillation period, though not highly significant, further contribute to the observed difference. The challenges in obtaining accurate measurements during the experiment are highlighted by these factors altogether.

The study's findings on gravitational acceleration measurement using a mathematical pendulum show both consistencies and variations when compared to previous research. A value of 9.95974 m/s² was obtained from this experiment, which is slightly higher than the normal gravitational acceleration of 9.807 m/s². Similar deviations were observed in Amedeker's (2022) study, which reported a value of 9.93 m/s²; in both cases, unaccounted factors like air resistance and string mass likely influenced the results. Unlike these results, Oliveira's (2016) experiment showed a more precise measurement of 9.81 m/s² by carefully factoring in the string's weight, showing that proper consideration of physical limitations leads to more reliable data. These variations underscore the substantial influence of experimental design choices and underlying assumptions on the obtained outcomes.

The experimental measurements were affected by multiple critical variables. Most notably, omitting the string's mass from calculations produced systematic errors, as demonstrated by the higher values obtained compared to Oliveira's corrected measurements. Air resistance influenced the results by damping the pendulum's oscillations and modifying its expected motion. Furthermore, manual measurements of both string length and period duration introduced slight inaccuracies. These combined effects illustrate the inherent difficulties in achieving experimental results that precisely align with theoretical predictions, despite rigorous control measures.

Findings confirmed the fundamental physical principle where longer pendulum strings produce proportionally longer swing periods, with the period squared (T^2) showing direct proportionality to length (l). This aligns with findings from Rivera-Figueroa & Lima-Zempoalteca (2021), though their use of digital sensors provided greater precision by reducing human error. Consistent with Sardjito and Yuningsih's (2020) findings on amplitude dependence, the study maintained angular displacements below 10° to preserve the small-angle approximation required for valid harmonic motion analysis.

To enhance measurement accuracy in future experiments, several improvements could be made. As demonstrated by Oliveira, incorporating string mass into calculations would eliminate a significant source of error. Reducing air resistance through vacuum environments or lighter materials could minimize damping effects. Implementing digital measurement tools utilized by Fauzi et al. (2024) would increase timing precision. Furthermore, considering geographical variations in gravitational acceleration could lead to a deeper understanding of the findings. These refinements would build on the current study's findings while addressing its limitations.

The differences between this study's results and previous research primarily stem from three areas: unaccounted physical factors in the experimental setup, limitations of manual measurement techniques, and variations in methodological approaches. Although the pendulum technique maintains its theoretical validity for determining gravitational acceleration, these comparisons show how careful attention to experimental details and conditions can significantly affect outcomes. Future studies can provide even more precise measurements of this fundamental physical constant by applying validated approaches from previous studies to address experimental imperfections and enhance measurement protocols.

CONCLUSION

The study concluded that a simple mathematical pendulum setup can effectively measure gravitational acceleration, yielding a value of approximately 9.95974 m/s^2 by analyzing the relationship between the regression function's gradient and the period's general equation. It was also found that the square of the period (T^2) is directly proportional to the string's length ($T^2 \propto l$), indicating that shorter strings result in faster pendulum movement while longer strings increase oscillation periods. Additionally, maintaining an initial deviation angle below 10 degrees ($\theta \leq 10^\circ$) was crucial for obtaining precise gravitational acceleration estimates. To enhance future studies, several recommendations were proposed, including a thorough understanding of theoretical concepts, ensuring an interference-free experimental setup, meticulous measurement of string length, load distance, and oscillation duration, increasing the number of calculations per trial, and keeping the initial deviation angle below 10 degrees to align results more closely with theoretical expectations.

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