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Calculation of Annual Premiums and Premium Reserves for Endowment Joint Life Insurance Based on Stochastic Interest Rates Using the Monte Carlo Method

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This study examines the determination of annual premiums and premium reserves for an endowment joint life insurance product by incorporating interest rate uncertainty through the Cox-Ingersoll-Ross (CIR) stochastic model and Monte Carlo simulation. The Indonesian Mortality Table 2023 is used to compute joint survival probabilities for the three insured individuals, while the CIR parameters are estimated from historical interest rate data for the period 2020-2024. The present value of benefits and annuities is calculated along each simulated path, enabling the premium and premium reserves to be evaluated prospectively based on fluctuating interest rate dynamics. The results show that the magnitude of premiums and reserves is influenced by the initial ages of the insured, the mortality structure, the sum assured, and the variability of the simulated interest rates. At the beginning of the contract, all scenarios produce negative reserves because accumulated premiums are still insufficient to cover the expected present value of benefits. However, the reserves increase steadily over time and turn positive toward the end of the insurance term. These findings indicate that the Monte Carlo approach based on the CIR model provides a more adaptive and realistic representation of premium and reserve behavior compared with deterministic methods, thereby supporting more accurate financial risk assessment for insurance companies.
Keywords: Endowment Joint Life Insurance, Premium Reserve, Monte Carlo, CIR Model, Stochastic Interest Rate.

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1. Introduction

Insurance serves as a financial mechanism designed to provide protection against future risks, relying on various financial components to maintain its sustainability and stability. One of the key elements in insurance is the premium the amount paid by policyholders to the insurer under specified terms, as a prerequisite for coverage (Nadiyyah, 2024). Among the common premium structures, annual premiums are typically paid at the beginning of each policy year, either as a fixed or variable amount, depending on policy terms. The calculation of a net annual premium is derived from the combination of a single net premium and the annuity value at the beginning of the payment period (Hasriati, 2024). In addition to premiums, insurance companies allocate funds in the form of premium reserves, which are portions of collected premiums set aside to ensure the company's financial capacity to meet future claims, functioning as a safeguard against potential claim risks (Kele, 2019).

Insurance plays a crucial role in mitigating financial risks by providing compensation for potential losses, particularly through life insurance, which guarantees payment to beneficiaries upon the insured's death. Life insurance can take several forms, including term, whole life, and endowment policies (Mhd Ghazy Arkan Indah Cantika Balqis Muhammad Fakhrurrozy Siregar, 2024). The endowment type is highly favored as it not only offers protection but also functions as a savings or investment instrument, providing benefits in the form of payouts upon the insured's death during the protection period or upon policy maturity, with the

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possibility of early claims based on contractual terms (Fikri, 2022). The computation of annual premiums and premium reserves in endowment life insurance is heavily influenced by stochastic interest rates that reflect market uncertainty, with models such as Vasicek and Cox Ingersoll Ross providing more realistic simulations compared to deterministic approaches. In joint life endowment policies, the calculation becomes more complex as it incorporates the joint survival probabilities of two individuals, adding further actuarial challenges (Satya Widjaja, 2023).

The Monte Carlo method, a random-number-based simulation technique, is effective for estimating annual premiums and premium reserves under uncertain market conditions. This process involves defining mathematical models, generating random variables following specified probability distributions, and repeating simulations to obtain statistically accurate estimates. Monte Carlo simulations are widely used in risk analysis and financial modeling. Rahmada Putri (2023) demonstrated that this method effectively quantifies cost uncertainty and produces more realistic estimates under fluctuating market conditions, making it a vital tool in the insurance industry.

Various approaches have been developed to optimize premium-setting strategies in insurance markets characterized by incomplete information. The Bayesian Nash equilibrium technique, for example, has been used to analyze interactions among insurers when setting premiums. Connery (2022) explored life insurance premium computation under Sharia principles using Brownian motion and Monte Carlo simulations, showing that estimated premiums are comparable to conventional methods while remaining Sharia-compliant. Similarly, Gao (2023) applied a parametric expectile regression approach to automobile insurance, revealing that extreme claims significantly influence premium determination and introducing the Expectile Premium Principle for more efficient premium allocation. Moreover, Mourdoukoutas (2024) emphasized the impact of insurers' risk-aversion levels on premium-setting strategies within competitive markets.

The data used in this study include stochastic historical interest rates and mortality data from life tables to calculate joint life survival probabilities. Interest rate data were obtained from financial reports or secondary sources from financial institutions for the period 2020–2024, while mortality data were taken from actuarial tables published by official agencies. Using the Monte Carlo method, this study simulates various stochastic interest rate scenarios to produce more accurate estimates of annual premiums and premium reserves.

This research modifies the deterministic approach proposed by Miasary (2023), who calculated annual premiums and premium reserves for joint life endowment insurance under stochastic interest rates. The modification involves applying the Monte Carlo method, which better handles market uncertainty and interest rate volatility. Monte Carlo simulations were chosen for their ability to generate realistic stochastic scenarios, leading to more accurate estimates of premiums and reserves than deterministic models (Miasary, 2023).

2. Method

This study employs a quantitative approach using a simulation-based method to calculate annual premiums and premium reserves for endowment joint life insurance under stochastic interest rates. The data used consist of historical interest rate data for the period 2020–2024 obtained from secondary financial sources, as well as mortality data from official life tables used to determine joint life survival probabilities.

Stochastic interest rates are modeled using a mean-reverting continuous-time framework, with parameters estimated from historical data. Monte Carlo simulation is applied to generate multiple interest rate paths, which are then used to calculate discount factors, present values of insurance benefits, and annuity values. Annual premiums are determined based on the equivalence principle, while premium reserves are calculated prospectively for each policy year. Final estimates are obtained by averaging the simulation results to reflect interest rate uncertainty more realistically than deterministic approaches.

3. Results and Disccusion

Mortality and Interest Rate Data

This study used the 2023 Indonesian Mortality Table and annual interest rate data (2020–2024) to estimate parameters of the Cox Ingersoll Ross (CIR) stochastic interest rate model. Estimated parameters were:

$$c = 0.3991, \qquad \theta = 0.0609, \qquad \sigma = 0.0893.$$

Three joint-life endowment insurance scenarios were analyzed: Young (ages 30–28–5), Adult (45–43–15), and Pre-Retirement (58–56–33), each with a 20-year contract term.

Joint-Life Survival and Death Probabilities

Joint-life survival probability for three insured individuals (x, y, z) over k years is:

$$_{n}p_{xyz} = {_{n}p_{x}} \cdot {_{n}p_{y}} \cdot {_{n}p_{z}}$$

while the probability that at least one dies before year $k$is:

$$_{n}q_{xyz} = 1 - {_{n}p_{xyz}}$$

Results show that survival probability decreases with contract duration and age, with the steepest decline in the Pre-Retirement scenario due to higher initial mortality rates.

Table 1. Probability of Survival and Minimum of One Death

Year	Young		Adult		Pre-Retirement	
	p	*q*	*p*	*q*	*p*	*q*
0	0,99690	0,00310	0,99243	0,00757	0,97483	0,02517
10	0,99479	0,00521	0,98032	0,01968	0,95291	0,04709
19	0,98862	0,01138	0,96300	0,03699	0,92581	0,07419

Monte Carlo Simulation of the CIR Model

Interest rates were simulated using 1,000 Monte Carlo paths with Euler–Maruyama discretization:

$$r_{t+1} = r_t + c(\theta - rt)\Delta t + \sigma\sqrt{r_t}\sqrt{\Delta t}Z_t, \qquad Z_t \sim \mathcal{N}(0,1).$$

The average simulated interest rate remained around 6.1%, exhibiting *mean reversion* toward the long-term mean θ.

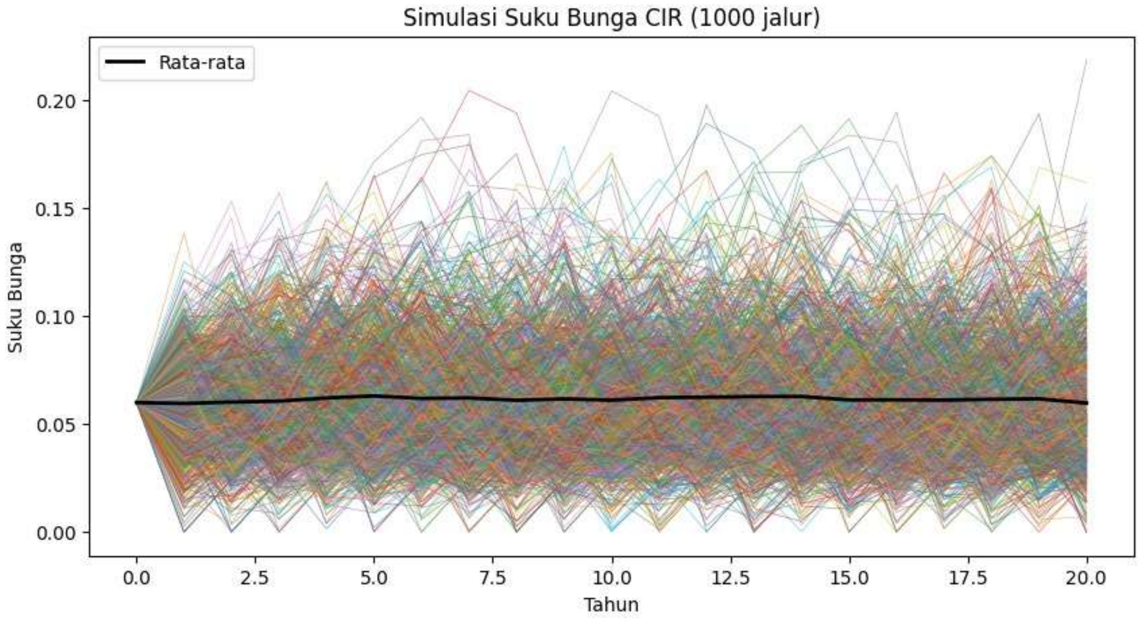

Figure 1. 1000 Interest Rate Simulation Paths

Present Value of Benefits and Annuities

Using expected interest rates from the CIR model, the present value of benefits $A_{xyz:\bar{n}|}$ and the annual annuity value $\ddot{a}_{xyz:\bar{n}|}$ were computed as:

$$A_{xyz:\bar{n}|} = \sum_{k=0}^{n-1} v^{k+1} \cdot {}_{k}q_{xyz} + v^{n} \cdot {}_{n}p_{xyz},$$

$$\ddot{a}_{xyz:\bar{n}|} = \sum_{k=0}^{n-1} v^{k} \cdot {}_{k}p_{xyz}$$

Where $v^t = \frac{1}{1+r_t}$.

Table 2. Present Value of Benefits and Annuities

Scenario	A	$\ddot{a}$
Young	1.0407	18.7313
Adult	1.2834	18.4644
Pre-Retirement	1.7632	17.9496

Higher age corresponds to higher benefit values and lower annuities due to increased mortality risk.

Annual Premium Calculation

The annual net premium per unit sum insured is calculated as:

$$P = \frac{A_{xyz:\bar{n}|}}{\ddot{a}_{xyz:\bar{n}|}}$$

Table 3. Annual Premium

Scenario	P
Young	0.05556
Adult	0.06951
Pre-Retirement	0.09823

Premiums increase with age because of higher mortality and smaller expected annuity values.

Premium Reserve

The prospective reserve at time $t$is:

$$\boldsymbol{v_t} = A_{xyz+t:n-t} - P \cdot \ddot{a}_{xyz+t:n-t}$$

Reserves were computed annually for each scenario. Initially, reserves are negative—reflecting higher early-year risk—but gradually increase and become positive as more premiums accumulate.

Table 4. Premium Reserves

Year	Young	Adult	Pre-Retirement
0	-0.274	-0.305	-0.354
10	0.192	0.218	0.218
20	0.989	0.963	0.926

For an insured amount (UP) of IDR 200,000,000, the final reserves reach approximately IDR 197.7 million (Young), IDR 192.6 million (Adult), and IDR 185.2 million (Pre-Retirement).

Interpretation

Simulation results confirm that higher initial age yields higher annual premiums and lower final reserves. The CIR-based stochastic interest model captures market uncertainty effectively, with Monte Carlo simulations providing realistic estimates for long-term premium and reserve dynamics in joint-life endowment insurance.

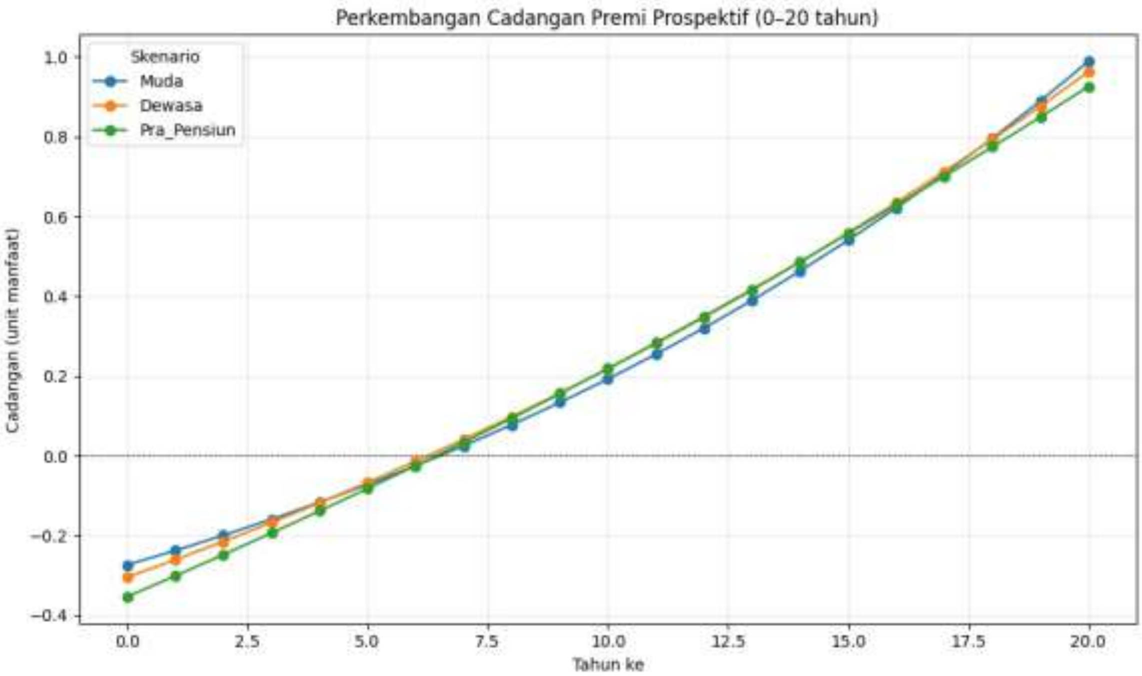

Figure 2. Premium Reserve Comparison

4. Conclusion

This study analyzed the determination of annual premiums and premium reserves for joint-life endowment insurance using the Monte Carlo simulation method under a stochastic interest rate framework based on the Cox Ingersoll Ross (CIR) model. The results show that annual premiums depend on the joint-life survival probability, the present value of benefits, and annuity factors derived from simulated interest rate paths. Premiums increase with the initial age of the insured, reflecting lower survival probabilities and shorter premium payment durations. The prospective premium reserve follows a consistent pattern across all age

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scenarios starting low at the beginning of the contract and increasing gradually as premiums accumulate. Although stochastic interest rate variations cause slight differences in reserve values, the overall pattern remains stable. Adjusting the sum assured affects only the scale of reserves without altering the underlying structure. Overall, the Monte Carlo approach with the CIR stochastic interest rate model provides a realistic representation of benefit and premium dynamics under interest rate uncertainty, supporting more accurate and flexible actuarial decision-making for life insurance companies.

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