Application of Robust Regression with M-Estimation in the Analysis of the Human Development Index in Papua Province

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The Human Development Index (HDI) is an important indicator for measuring human development achievements in terms of health, education, and a decent standard of living. In Papua Province, substantial differences across regions may produce extreme observations that can affect the stability of Ordinary Least Squares (OLS) estimates. This study examines the statistical relationships between the HDI and four of its component indicators in Papua Province, identifies influential observations, and applies robust regression with M-estimation (Huber function) as an alternative to OLS. Secondary data from Statistics of Papua Province covered nine regencies/cities over 2021–2025 (45 regency-year observations), analyzed as a pooled regression. The dependent variable was the HDI, and the independent variables were Life Expectancy, Mean Years of Schooling, Expected Years of Schooling, and Per Capita Expenditure. The analysis comprised OLS estimation, classical assumption tests, Weighted Least Squares (WLS) to address heteroscedasticity, and Huber M-estimation using the Iteratively Reweighted Least Squares (IRLS) algorithm with the WLS weights as prior weights. The OLS residuals violated the assumptions of normality and homoscedasticity. Studentized residuals flagged two observations (Mamberamo Raya, 2021–2022) as residual outliers, neither of which was a high-leverage point, and ten observations received Huber weights below 1. In the Huber model, Life Expectancy, Expected Years of Schooling, and Per Capita Expenditure were statistically significant at the 5% level, whereas Mean Years of Schooling was significant only at the 10% level ($p = 0.0994$), which is weaker evidence. Because the predictors are components of the HDI, the results are interpreted as statistical associations rather than causal effects. The robust regression results indicated that Life Expectancy, Expected Years of Schooling, and Per Capita Expenditure significantly affected HDI at the 5% significance level, while Mean Years of Schooling was significant at the 10% level. Per Capita Expenditure and Expected Years of Schooling were the most statistically significant factors. The Huber robust regression model produced a Residual Standard Error (RSE) of 0.9373 and effectively reduced the influence of outliers, resulting in more stable and representative parameter estimates of human development conditions in Papua Province.

Keywords: Human Development Index, Robust Regression, M-Estimation, Huber, Outliers, Papua Province.

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1. Introduction

The Human Development Index (HDI) is used to describe a region's development achievements in terms of health, education, and a decent standard of living. This measure offers a broader perspective than economic indicators alone, as development also encompasses opportunities for the population to access education, lead healthy lives, and meet daily needs. Statistics Indonesia (BPS) calculates the HDI using indicators such as life expectancy at birth, expected years of schooling, mean years of schooling, and adjusted real per capita expenditure [1], [2]. In the context of Papua Province, the HDI

serves as a crucial indicator for assessing the evolution of the population's quality of life and determining whether development progress has been distributed equitably across its various regencies and cities.

Regional characteristic variations are a key factor to consider when analyzing the HDI of Papua Province. Geographical conditions, the availability of educational and healthcare facilities, employment opportunities, and the economic capacity of the population can vary significantly from one area to another. These disparities must be examined through data so that development assessments do not rely solely on the province's aggregate HDI figure. Research by Syafri et al. [3] explores the relationships between poverty, unemployment, inflation, locally-generated revenue, and foreign investment with the HDI at the provincial level in Indonesia. Meanwhile, Fadhil et al. [4] employed panel data regression to identify variables associated with the HDI in Riau Province. These studies demonstrate that statistical analysis can be used to examine the link between human development outcomes and socio-economic conditions, provided that regional characteristics are taken into account.

Linear regression is one approach used to measure the relationship between a response variable and several explanatory variables. In the Ordinary Least Squares (OLS) method, regression coefficients are derived by minimizing the sum of squared residuals. However, observations with extreme values can exert a significant influence on estimation results; consequently, model coefficients may shift depending on whether specific observations are included or excluded. De Menezes et al. [5] outline various M-estimators developed to address such conditions. Björklund et al. [6] also discuss robust regression approaches designed to ensure that estimates do not rely too heavily on observations containing large errors. Consequently, before a regression model is employed to draw conclusions regarding the Human Development Index (HDI), it is essential to examine the data characteristics and the potential presence of outliers.

In light of these considerations, this study applies robust regression with M-estimation to analyze the HDI in Papua Province. This approach was selected because it offers an alternative estimation method for situations where extreme residuals might otherwise influence regression results. Research by Damayanti and Susanti [7] compared Huber and Tukey Bisquare weight functions using Indonesian poverty data, while Sakinah et al. [8] evaluated various M-estimation weight functions using data on realized foreign direct investment. Although the subjects of these studies differ, the approaches employed provide a methodological foundation for research on the HDI. This study aims to identify observations that could potentially influence estimates, construct an M-estimation robust regression model based on selected socio-economic variables, and compare the model's results with those of Ordinary Least Squares (OLS) using appropriate evaluation metrics.

2. Literature Review and Problem Statement

The Human Development Index (HDI) is an indicator used to assess the quality of life of a population across the dimensions of health, education, and a decent standard of living. Research on the HDI in Indonesia has employed various statistical approaches to examine its relationship with socio-economic conditions. Syafri et al. [3] analyzed factors associated with the HDI across provinces, while Fadhil et al. [4] used panel data regression to study the HDI in Riau Province. Nisa and Maria [9] investigated factors related to the HDI in DKI Jakarta, whereas Tyas and Sukartini [10] analyzed the links between health facilities, education, unemployment, and socio-economic conditions and human development in Indonesia. Other studies have also utilized linear regression and multivariate approaches to examine human development indicators [11]–[14]. These studies establish a basis for analyzing the HDI through statistical modeling. However, the selection

of explanatory variables must take into account the theory and structural composition of the HDI to ensure that the analysis results do not merely replicate the mathematical relationships already inherent in the index.

Robust regression using M-estimation is an approach applicable when outliers may potentially influence regression estimates. Unlike Ordinary Least Squares (OLS), which minimizes the sum of squared residuals, M-estimation employs a loss function to regulate the contribution of residuals during the parameter estimation process. De Menezes et al. [5] reviewed various types of M-estimators and their characteristics, while Björklund et al. [6] developed a robust regression approach based on error tolerance. Damayanti and Susanti [7] compared Huber and Tukey Bisquare weighting functions in poverty analysis, whereas Sakinah et al. [8] compared several weighting functions using investment data. Other studies have addressed the development of robust estimators, outlier detection, and the comparison of various estimation methods [15]–[20]. Nevertheless, based on the literature used in this study, the application of M-estimation to the analysis of the Human Development Index (HDI) at the regency/city level in Papua Province warrants specific evaluation. The research focuses on identifying outliers, formulating an M-estimation model, and comparing the model's performance with that of Ordinary Least Squares (OLS). The methodological hypothesis posits that M-estimation has the potential to yield results that are more robust against extreme residuals—provided outliers are indeed influencing the OLS model—a hypothesis that must be verified using the research data.

The Huber weighting function was selected using four criteria. First, the Huber function is monotone: it gives a weight of 1 to observations whose scaled residual is within the tuning constant k and a weight of $\frac{k}{|u|}$beyond it, so no observation is rejected outright [20], [21]. Second, its loss function is convex, so IRLS converges to a unique solution regardless of the starting values, whereas redescending functions such as Tukey's bisquare and Andrews' sine (compared in [7], [8]) assign zero weight to large residuals and can have multiple local solutions [22]. Third, the sample is small (n = 45 with five parameters) and the extreme observations are published regional statistics that, unless shown to be recording errors, should remain in the analysis with reduced influence rather than be rejected. Fourth, with k = 1.345 the Huber estimator retains about 95% of the efficiency of least squares under normally distributed errors [21]. These criteria justify the Huber function as a conservative choice; they do not imply that it is superior to other functions. Because [7] and [8] compared several functions on different datasets, the choice should be justified for each dataset, and comparisons with redescending functions or MM-estimation [23] are treated as robustness checks for future work.

In regression, an unusual observation can be extreme in the response given the predictors (a residual or vertical outlier), extreme in the space of the predictors (a leverage point), or both; its influence on the fitted coefficients depends on the combination of the two [22], [24], [25]. Leverage is measured by the hat values h_{ii}, whose mean is $\frac{p}{n}$ and for which $\frac{2p}{n}$ is a common cut-off [24], and residual outliers are commonly identified by studentized residuals beyond ±2 [24]. A good leverage point (extreme predictors, small residual) does not harm the estimates, while a bad leverage point (extreme predictors and large residual) can distort them. The two types also call for different handling: M-estimators with a monotone ψ function, such as Huber's, bound the influence of large residuals but not the influence of leverage points, so high-breakdown methods such as MM-estimation are needed when bad leverage points are present [22], [23]. This study therefore classifies observations with both studentized residuals and hat values before interpreting the Huber weights.

3. Method

This study employs a quantitative explanatory approach that aims to examine the statistical relationships between selected socioeconomic indicators and the Human Development Index (HDI). Because the design

is observational and the predictors are themselves components of the HDI, the estimated coefficients are interpreted as conditional statistical associations and not as causal effects. The analysis applies classical (OLS and WLS) and robust (M-estimation) methods to compare the stability of the estimated models. The unit of analysis is the regency/city in Papua Province. The variables examined in this study consist of one dependent variable and four independent variables, as follows:

Dependent Variable (Y)

Y = Human Development Index (HDI) Independent (index on 0-100 scale)

Independent Variables (X)

X_1 = Life Expectancy (years)
X_2 = Mean Years of Schooling (years)
X_3 = Expected Years of Schooling (years)
X_4 = Expenditure per Capita (Thousand rupiah per person per year)

Data Structure and Sources

The data are secondary data from Statistics of Papua Province (BPS Provinsi Papua) for nine regencies/cities over 2021–2025 (9 × 5 = 45 regency-year observations). Although the observations form a panel (nine regencies observed over five years), they are analyzed as a single pooled sample of 45 observations with ordinary multiple linear regression (OLS), WLS, and Huber M-estimation; regency-specific and time-specific effects are not modeled. Repeated observations of the same regency are therefore treated as independent, which may understate standard errors if the errors are serially correlated. A fixed-effects or random-effects panel specification was not estimated.
The multiple linear regression model is formulated as:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \varepsilon$$

The model parameters are initially estimated using the Ordinary Least Squares (OLS) method, which minimizes the sum of squared residuals.

Classical Assumption Tests

Prior to conducting the regression analysis, several classical assumption tests are performed to evaluate the adequacy of the OLS model. These include:

1. Normality test, using the Shapiro–Wilk test complemented by a normal Q–Q plot;
2. Multicollinearity test, using variance inflation factors (VIF)
3. Heteroscedasticity test, using the studentized Breusch–Pagan test with WLS
4. Autocorrelation test, using the Durbin–Watson statistic.

Weighted Least Squares (WLS)

When the Breusch–Pagan test rejects constant error variance, the OLS estimator remains unbiased but is no longer efficient, and its usual standard errors are unreliable. WLS was therefore applied by minimizing $\sum v_i e_i^2$, with weights inversely proportional to the estimated error variance [29]. The weights were determined in three steps. First, the OLS residuals e_i and fitted values $\hat{y}_i$ were obtained. Second, the absolute residuals were regressed on the fitted values (a Glejser-type auxiliary regression [30]):

$$|e_i| = \gamma_0 + \gamma_1 \hat{y}_i + \nu_i$$

and the fitted values $\hat{s}_i$ of this regression were used as estimates of the error standard deviation of observation i. Third, the weights were set to

$$v_i = \frac{1}{\hat{s}_i^2}$$

This specification assumes that the error standard deviation changes linearly with the fitted HDI. Because the weights are estimated from OLS residuals, which may themselves be affected by outliers, and because their sampling variability is ignored in the standard errors, the WLS results are used for comparison. The weights v_i were then passed as prior weights to the M-estimation step: Huber M-estimation limits the influence of large residuals but does not by itself correct unequal error variances, so combining both sets of weights is intended to address the two problems together.

M-Estimation

To improve the robustness of the regression estimates in the presence of potential outliers or influential observations, this study also applies M-estimation, originally introduced by Peter J. Huber (1964) [20]. The letter M refers to the *maximum likelihood type* of estimation.

Unlike OLS, which minimizes the sum of squared residuals, M-estimation minimizes a more general loss function:

$$\min \sum_{i=1}^{n} \rho(e_i)$$

where ρ denotes the loss function and e_i represents the residual associated. The loss function used in M-estimation increases less rapidly than the squared loss for large residuals. Consequently, observations with unusually large residuals receive less influence in the estimation process than they would under the OLS approach. This characteristic makes M-estimation less sensitive to outliers.

In this study ρ is the Huber function [20], [21]. For the scaled residual $u_i = \frac{e_i}{\hat{s}}, where\ \hat{s} = \frac{\text{median}|e_i - \text{median}(e)|}{0.6745}$ is the normalized median absolute deviation, the loss function, the influence function ψ, and the weight function w are:

$$\rho(u) = \begin{cases} \frac{u^2}{2}, & \text{if } |u| \le k \\ k|u| - \frac{k^2}{2}, & \text{if } |u| > k \end{cases}$$

$$\psi(u) = \begin{cases} u, & \text{if } |u| \le k \\ k\,\text{sign}(u), & \text{if } |u| > k \end{cases}$$

$$w(u) = \frac{\psi(u)}{u} = \begin{cases} 1, & \text{if } |u| \le k \\ \frac{k}{|u|}, & \text{if } |u| > k \end{cases}$$

with tuning constant k = 1.345, which gives about 95% efficiency under normally distributed errors [21]. Observations with $|u_i| \le k$ keep a weight of 1; observations with $|u_i| > k$ are down-weighted in proportion to $\frac{k}{|u|}$ but are never excluded.

Iteratively Reweighted Least Squares (IRLS)

The M-estimation procedure is implemented using the Iteratively Reweighted Least Squares (IRLS) algorithm. The estimation process consists of the following steps:

Step 1: Initial Estimation

The initial regression coefficients are estimated by least squares (in this study, the WLS fit with the prior weights v_i).

Step 2: Calculation of Residuals

The residuals are calculated as:

$$e_i = y_i - \hat{y}_i$$

Step 3: Calculation of Weights

Weights are assigned to each observation based on the magnitude of its residual:

$$w_i = \frac{\psi(e_i)}{e_i}$$

For the Huber function, $w_i = w(u_i)$ as defined above. Observations with relatively large scaled residuals receive smaller weights, thereby reducing their influence on the estimated regression coefficients.

Step 4: Re-estimation

The regression coefficients are subsequently re-estimated using weighted least squares, where W is the diagonal matrix of the combined weights $v_i \cdot w_i$ (prior WLS weight times Huber weight):

$$\hat{\beta} = (X^T W X)^{-1} X^T W Y$$

Step 5: Iteration and Convergence

Steps 2–4 are repeated until the relative change in the residuals between two successive iterations falls below 10^{-4} (the default accuracy of the rlm function), with a maximum of 20 iterations. The final weights are those of the last iteration.

All computations were performed in R with the rlm function of the MASS package [31], using psi = psi.huber (k = 1.345), MAD-based scale estimation, the WLS weights passed through the weights argument, and the default convergence settings. Standard errors and t-values were taken from the rlm output, and two-sided p-values were computed from the t distribution with n – p = 40 degrees of freedom; this is an approximation because the sampling distribution of robust estimates is asymptotic.

Outlier and Leverage Diagnostics

Outliers were examined with OLS-based diagnostics before interpreting the Huber weights. An observation was classified as a residual (vertical) outlier if its absolute studentized residual exceeded 2. High-leverage observations were identified from the hat values (the diagonal of $H = X(X^T X)^{(-1)} X^T$) using the cut-off $(2p)/n = \big(2(5)\big)/45 = 0.222$ [24]. An observation is a bad leverage point if both criteria are met and a good leverage point if only the leverage criterion is met [22]. Separately, observations with Huber weights below 1 are described as down-weighted observations: a weight below 1 only means that the scaled residual exceeds k, and it is not in itself a formal test for outliers.

4. Results and Discussion

Descriptive Analysis of Research Data

This study analyzes the statistical relationships between the Human Development Index (HDI) and four of its component indicators in Papua Province using Huber M-estimation robust regression. The data have a panel structure of 9 regencies/cities over a five-year period (2021–2025), giving 45 observations, but they were analyzed as a single pooled sample. The dependent variable is the HDI (Y), and the independent variables are Life Expectancy (X1), Mean Years of Schooling (X2), Expected Years of Schooling (X3), and Per Capita Expenditure (X4).

Table.4.1. Summary of the IPM Data

Statistik	X1	X2	X3	X4	Y
Minimum	68.11	11.80	5.870	4,603	52.18
Kuartil 1 (Q1)	69.37	12.46	8.870	6,788	65.84
Median	69.94	12.85	9.430	8,091	68.78
Mean (Rata-rata)	70.06	13.29	9.454	8,815	69.22
Kuartil 3 (Q3)	70.72	14.24	10.340	10,229	72.85
Maksimum	72.34	15.28	12.080	15,522	82.26

The raw BPS data show large disparities in human development across regions. Kota Jayapura recorded the highest values, with an HDI of 82.26 and an adjusted per capita expenditure of Rp15,522,000 in 2025,

whereas Mamberamo Raya Regency recorded the lowest HDI (52.18) and the lowest per capita expenditure (Rp4,603,000) in 2021. Such wide dispersion makes extreme values plausible, but dispersion alone does not establish that an observation is an outlier in the regression sense; outliers were therefore assessed with residual and leverage diagnostics (Sections 4.3 and 4.5).

Estimation of the Classical Ordinary Least Squares (OLS) Model

The initial step in analyzing empirical parameters is carried out through classical linear fitting based on the Ordinary Least Squares (OLS) model. The regression equation obtained by running the data in the RStudio console is presented as follows:

```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -3.772e+01  3.382e+01  -1.115   0.2713
x1           1.119e+00  4.943e-01   2.263   0.0291 *
x2           2.342e-02  4.272e-01   0.055   0.9566
x3           2.064e+00  3.275e-01   6.301 1.78e-07 ***
x4           9.911e-04  1.618e-04   6.125 3.16e-07 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.2 on 40 degrees of freedom
Multiple R-squared:  0.9716,    Adjusted R-squared:  0.9688
F-statistic: 342.2 on 4 and 40 DF,  p-value: < 2.2e-16
```

Figure 4.1 OLS estimation results

$$\hat{Y} = -37,72 + 1,119\,X1 + 0,02342\,X2 + 2,064\,X3 + 0,0009911\,X4$$

Based on the OLS model above, partial significance tests (t-tests) at the 5% level show that Expected Years of Schooling (X3) and Per Capita Expenditure (X4) are statistically significant, with the smallest p-values (1.78e-07 and 3.16e-07, respectively). Life Expectancy (X1) is also significant (p = 0.0291), whereas Mean Years of Schooling (X2) is not (p = 0.9566). The OLS residuals range from −3.4365 to 1.7087; the large negative minimum shows that at least one observation lies far below the fitted plane. Large residuals can inflate the residual variance on which the standard errors are based, which is one possible reason for the imprecision of the X2 estimate, but this explanation was not tested here. Collinearity between the two schooling indicators (X2 and X3) is another plausible contributor and is considered in Section 4.3.

Classical Assumption Testing and Outlier Detection

To verify the validity of the interpretation of the OLS model significance test, adherence to the BLUE (Best Linear Unbiased Estimator) criteria is assessed through normality diagnostic procedures and a heteroskedasticity test.

Multicollinearity and autocorrelation.

```
        Durbin-Watson test

data:  OLS
DW = 1.8037, p-value = 0.2449
alternative hypothesis: true autocorrelation is greater than 0

> #Tidak Ada Multikol
> vif(OLS)
      x1       x2       x3       x4
6.710631 6.272234 7.213711 6.795600
```

Figure 4.2 Durbin Watson test and VIF test Result

Normality Test (Shapiro-Wilk and Q-Q Plot)

A formal test for the normality of OLS residuals was conducted using the Shapiro-Wilk criterion. A test statistic of W = 0.93078 was obtained, with a p-value of 0.009992. Based on the rejection criterion of p < 0.05, the decision was made to reject the null hypothesis (H0). This leads to the conclusion that the residuals from the OLS regression model are not normally distributed.

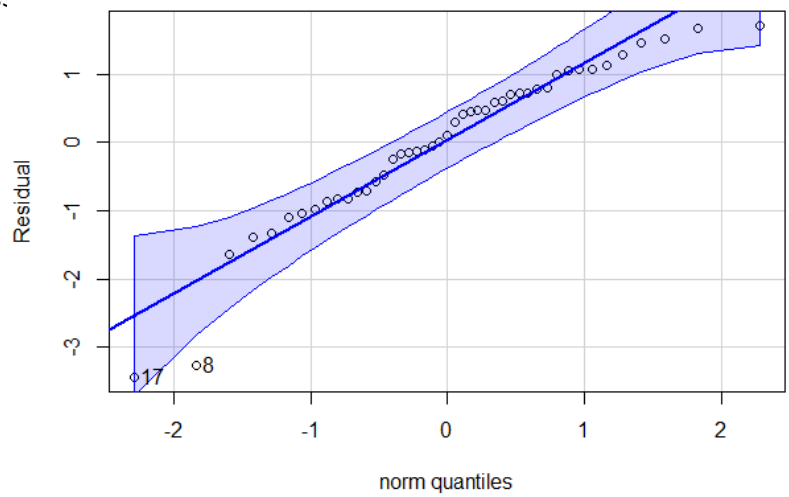


Figure 4.3 Normal Q-Q Plot Diagram

Visually, the Normal Q-Q plot indicates non-normality in the data. Although the data points ranging from the center to the upper right align neatly along a linear path, there is a sharp curvature at the lower end that extends outside the blue zone (representing the 95% confidence interval). The system identifies data points 8 and 17 as outliers.

Heteroskedasticity Test (Breusch-Pagan)

The homoskedasticity assumption was tested using the Studentized Breusch-Pagan test. The calculations yielded a BP statistic of 19.896 with 4 degrees of freedom (df) and a p-value of 0.0005235. Since the p-value is well below the critical threshold of 0.05, the null hypothesis (H0)—which posits constant residual variance—is rejected. Consequently, the OLS regression model exhibits a heteroskedasticity problem.

Application of Weighted Least Squares (WLS)

To address heteroscedasticity, the model was re-estimated by WLS using the weights described in Section 3 ($v_i = \frac{1}{\hat{s}_i^2}$, where $\hat{s}_i$ is obtained from the regression of the absolute OLS residuals on the fitted values). A Breusch–Pagan test applied to an auxiliary regression of the squared weighted residuals on X1 and X2 only (df = 2) gave a statistic of 3.6968 (p = 0.1575), so no evidence of remaining heteroscedasticity related to X1 and X2 was found. This check is limited: it uses only two of the four regressors and is not the standard test of the weighted residuals against all regressors, and it was applied to the WLS model and not to the final Huber model. We therefore do not claim that heteroscedasticity has been resolved in the final Huber model.

Robust Regression Estimation via M-Estimation (Huber)

To limit the influence of extreme residuals, Huber M-estimation was applied with the WLS weights supplied as prior weights (Section 3). Observations whose scaled residual $|u_i|$ exceeds k = 1.345 are down-weighted by $k/|u_i|$ through the IRLS iterations until convergence; no observation is deleted. The coefficients of the three models are compared in Table 4.2.

Table.4.2. Summarizes the results across the OLS, WLS, and Robust Huber models.

Variable Indicator	OLS Estimate	WLS Estimate	Huber Robust Estimate	t-value (Robust)	p-value (Robust)	Significance
Constant (Intercept)	-37.7200	-32.9500	-41.9072	-1.4649	0.1508	Not Significant
X1 (Life Expectancy)	1.1190	1.0280	1.1658	2.7897	0.0080	Significant (α=1%)
X2 (Mean Years of Schooling)	0.0234	0.3328	0.6330	1.6870	0.0994	Significant (α=10%)
X3 (Expected Years of Schooling)	2.0640	1.9020	1.4045	4.3361	0.0001	Significant (α=1%)
X4 (Per Capita Expenditure)	0.00099	0.00088	0.0009	6.8772	<0.0001	Significant (α=1%)
Residual Std. Error (RSE)	1.2000	—	0.9373	—	—	df = 40

Outlier Analysis

Huber weights below 1 were assigned to 10 of the 45 observations (Table 4.3.). A weight below 1 means only that the scaled residual of the observation exceeds k = 1.345 (w = k/|u|); it is a down-weighting rule and not a formal test for outliers. Formal diagnostics from the OLS model (Figure 4.3) flagged only observations 8 and 17 (Mamberamo Raya, 2021 and 2022), whose studentized residuals are below −2 (approximately −3.4 and −3.6). Their hat values (approximately 0.21 and 0.17) are below the cut-off of 0.222, and no observation exceeds this cut-off. The two flagged observations are therefore residual outliers and not high-leverage points, and none of the 45 observations is a high-leverage point.

Table 4.3. Observations with Huber weights below 1 (observation numbers refer to the rows of the data set)

Obs.	Year	Regency/City	HDI (Y)	Huber weight	Classification
8	2021	Mamberamo Raya	52.18	0.2932	Residual outlier
17	2022	Mamberamo Raya	53.10	0.2659	Residual outlier
19	2023	Kota Jayapura	74.74	0.7036	Down-weighted only
28	2024	Kota Jayapura	75.32	0.8135	Down-weighted only
45	2025	Kota Jayapura	82.26	0.8873	Down-weighted only
7	2021	Waropen	62.72	0.8980	Down-weighted only
16	2022	Waropen	63.65	0.8811	Down-weighted only
21	2023	Biak Numfor	69.25	0.8601	Down-weighted only
32	2024	Biak Numfor	69.87	0.8973	Down-weighted only
37	2025	Keerom	76.14	0.9575	Down-weighted only

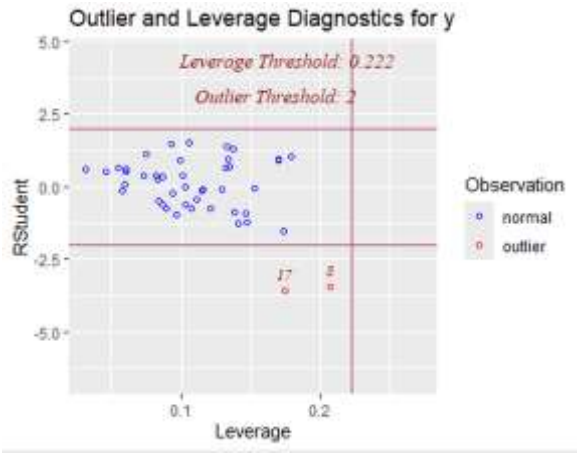


Figure 4.4 Studentized residuals versus leverage for the OLS model (cut-offs: |studentized residual| = 2 and leverage = 0.222). Observations 8 and 17 are flagged as outliers.

1. Residual outliers (Mamberamo Raya, 2021–2022): observations 8 and 17 have HDI values far below those predicted by the OLS model and received the lowest Huber weights (w8 = 0.2932, w17 = 0.2659). Their leverage is below the cut-off, so they are not high-leverage points. The data used here do not identify why these observations deviate; explanations such as geographic isolation or post-pandemic economic conditions are plausible contextual hypotheses that were not tested in this study.
2. Mildly down-weighted observations (Kota Jayapura 2023–2025, Waropen 2021–2022, Biak Numfor 2023–2024, and Keerom 2025): these eight observations received weights between 0.7036 and 0.9575 because their scaled residuals are moderately larger than k. They are not flagged by the studentized-residual or leverage criteria, so their weights are not formal evidence of outliers, and Kota Jayapura should not be described as a leverage point on this basis. Because the Huber function bounds only the influence of the residual, it gives no protection against bad leverage points; none was detected here, so this limitation does not appear to be binding, but a high-breakdown estimator such as MM-estimation [23] would be a useful robustness check.

The coefficient of X2 (Mean Years of Schooling) rose from 0.0234 (OLS) through 0.3328 (WLS) to 0.6330 (Huber), and its p-value changed from 0.9566 to 0.0994 (Table 4.2). This change should be interpreted cautiously. The Huber coefficient lies about 1.4 OLS standard errors from the OLS coefficient (difference 0.61; OLS standard error 0.4272), so the two estimates are statistically compatible. The standard error implied by the Huber estimate (0.6330/1.6870 ≈ 0.375) is only slightly smaller than the OLS standard error, so the change in significance reflects mainly a shift in the point estimate and not a gain in precision. The coefficient of X3 moved in the opposite direction (from 2.0640 to 1.4045), which suggests that the effect shared by the two schooling indicators is distributed differently between X2 and X3 depending on which observations are down-weighted, consistent with possible collinearity between them.

Interpretation of the Huber Robust Coefficient

In the Huber model, Expected Years of Schooling (X3, p = 0.0001), Per Capita Expenditure (X4, p < 0.0001), and Life Expectancy (X1, p = 0.0080) are statistically significant at the 5% level, and Mean Years of Schooling (X2, p = 0.0994) is significant only at the 10% level, which is weaker evidence than significance at 5%. All four coefficients are positive: higher values of each indicator are associated with a higher HDI, holding the other indicators constant. Because X1–X4 are the indicators from which the HDI is constructed, these coefficients reflect statistical relationships that are partly mechanical and should not be read as evidence that the variables cause changes in the HDI. The Huber RSE (0.9373) is smaller than the OLS RSE (1.2000), but the two are based on different estimation criteria and weights and cannot by themselves show that the robust model is more accurate; a fair comparison would require, for example, out-of-sample

prediction errors or estimates under alternative robust methods. The Huber model is therefore presented as an alternative whose estimates are less influenced by extreme residuals, not as the optimal model.

Limitations. (i) The data are analyzed as a pooled sample, ignoring regency-specific and time-specific effects. (ii) The WLS weights are estimated from OLS residuals and their sampling variability is ignored. (iii) Diagnostics of the final Huber model (for example plots of weighted residuals or tests on the scaled residuals) are not reported, so statements about normality and homoscedasticity are limited to the OLS and WLS results shown. (iv) The p-values of the robust model are approximate. (v) Other weighting functions and high-breakdown estimators were not compared.

5. Conclusion

This study applied OLS, WLS, and Huber M-estimation to 45 regency-year observations from nine regencies/cities in Papua Province (2021–2025), analyzed as a pooled sample. Because the four predictors are components of the HDI, the results describe statistical associations and not causal effects. In the Huber model, Per Capita Expenditure (X_4), Expected Years of Schooling (X_3), and Life Expectancy (X_1) were statistically significant at the 5% level, with X_4 and X_3 having the smallest p-values, whereas Mean Years of Schooling (X_2) was significant only at the 10% level ($p = 0.0994$), which is weaker evidence than significance at 5%; its coefficient was also unstable across the OLS (0.0234), WLS (0.3328), and Huber (0.6330) estimates. Formal diagnostics flagged two residual outliers (Mamberamo Raya, 2021–2022), neither of which was a high-leverage point, and eight further observations received only mild down-weighting. The Huber procedure yielded estimates that are less influenced by extreme residuals; however, this study did not show that estimation bias was eliminated or that violations of the OLS assumptions were fully resolved, and the RSE of the Huber model (0.9373) is not directly comparable with that of OLS. Future research should combine robust estimation with panel specifications, compare other robust estimators (for example MM-estimation), and report complete diagnostics for the final model.

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